

Research article

MODELING FLOW RATE INFLUENCED BY VOID RATIO AND POROSITY IN HOMOGENEOUS SILTY FORMATION THROUGH CROSS SECTION AREA IN A VERTICAL COLUMN

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Abstract

Modeling flow rate of fluid has been done by several experts around the globe, the concept may have develop result that may be better in the monitoring of flow rate of fluid in soil, but slight variation observed may be as a result of structural deposition of the soil matrix that may be seen in physical process as homogeneous formation, while in real condition it is not in structural stratifications of the formations, this inequalities in some formation of the soil are observed to developed slight variations of fluid flow rate in the soil structural deposition of the formations. The study observed void ratio and porosity of the soil as one of the influential parameters that pressured various variation of fluid flow rate in the soil formations, base on these observations, mathematical approach were found suitable to model the flow rate of fluid in vertical column of soil formation, the derived solution are developed base on the parameters that showcase to influences fluid flow migration under the law of plug flow application in soil and water environments. The expressed mathematical equations were derived to generate the final model that can be applied to monitor the flow rate of fluid in vertical column.

Keywords: modeling flow rate, void ratio and porosity, silty formation and vertical column

1. Introduction

More than last few decades worsening of both the quality and quantity of groundwater has become a universal phenomenon, which will further make stronger demand for drinking Water increases (World Bank report, 2006). Several severe cases of groundwater pollution with allusion to storm water infiltration have been documented worldwide (Lopes and Bender, 1998; Fischer, 2003). Few studies have also been documented nationally on groundwater with reference to main ions, trace elements and bacteriology (Arif 2007, Rahman et. al., 1997; Zubair, 1998, Liu and Li, 2005). However literatures are soundless on the impact of storm water permeation into groundwater. In recent years attention on the increasing ionic concentration of traces metals in groundwater as result of storm water infiltration has been studied by various workers (Ku et. al., 1992; Appleyard, 1993; Wild, 1994; Hathhorn and Yonge, 1995; Pitt, 1996; Lopes and Bander, 1998; Fisher, 2003). These have been ascribed to human Intervention, propagation of industries and recent agriculture practices in urban areas where storm water flow recharges the aquifer system and thus degrading the water quality. Contaminants present in urban storm water include volatile organic compound, pesticides, nutrients, and trace elements (Fisher, 2003). This can originate at the land surface or in the atmosphere (Lopes and Bender, 1998) Some constituents either volatize during storage or sorbs to the particulate matter (Michelson et. al., 1996) Arif 2007) and are not transported to the water table; however, are more persistent, and may threaten groundwater quality. Malmquist & Hard (1981) studied the impacts from sub surface infiltration at three sites in Sweden and concluded that storm water infiltration affects the groundwater quality to a small extent.

2. Theoretical background

The rate void ratio deposition has been found to influence the rate fluid flow in soil, this development can be expressed through laboratory experiment that determined void ratio in soil, similar condition found in the area of soil porosity, the rate of porosity in homogeneous silty formation are determine through the same laboratory investigation , since void ratio and porosity in homogeneous silty soil formation has a concrete relationship, it implies that the rate of flow will definitely be influenced by these two parameters in the system. Therefore stratification of the formation in vertical column has been found to be pressured by the two parameters in the formation, such condition were found to vary in several area where fluid flow are monitored in soil and water environment. These conditions are to examined the rate of fluid flow within the intercedes of the flow or flow path, either longitudinal or tortuosity flow in the soil , the rate of flow are base on the structural deposit of the soil, but the focus of these studies are base on the influences of void ratio and porosity deposit in soil and water environment, the rate of fluid flow in soil develop hydrostatic pressure of capillarity rise in either in construction of structures, it can also be observed on ground water where hydrostatic pressure can be observed through the rate of over burden pressure in the structural deposition of the formation. The expressed mathematical equations are base on the flow of fluid in vertical column under the influences of degree of void ratio and porosity of the soil. Subject to this relation, the rate of flow should be monitored to determine the variation of fluid flow through the degree of void ratio and

porosity in the strata, the derived mathematical expression will definitely expressed the rate of fluid flow variation at different formation, the concept will also give an insight of solute migration through the same similar sources in the study area.

The successful design and application of groundwater lowering methods depends not just on the nature of the groundwater environment (such as where the site is within the hydrological cycle), but is also critically influenced by the geology or structure of the soils and rocks through which water flows. This is especially true when trying to assess the effect of groundwater conditions on the stability of engineering excavations. The total hydraulic head at a given point is the sum of the 'pressure head' and the 'elevation head' at that point, The elevation head is the height of the measuring point above an arbitrary datum, and the pressure head is the pore water pressure u , expressed as metres head of water. Total hydraulic head is important because it controls groundwater flow. Water will flow from high total hydraulic head to low total hydraulic head. It follows that water does not necessarily flow from high pressure to low pressure or from high elevations to low elevations. In the presentation of his equation Darcy left no doubt of its origin being empirical. His important contributions to scientific knowledge were based on careful observation in the field and in the laboratory, and on the conclusions that he drew from these. Permeability is in fact only a theoretical concept, but one vital to realistic assessments of groundwater pumping requirements and so an understanding of it is most desirable. In theory, permeability is the notional (or 'Darcy') velocity of flow of pore water through unit cross sectional area. In fact, the majority of the cross sectional area of a soil mass actually consists of soil particles, through which pore water cannot flow. The actual pore water flow velocity is greater than the 'Darcy velocity', and is related to it by the soil porosity n (porosity is the ratio of voids, or pore space, to total volume). The main condition for Darcy's law to be valid is that groundwater flow should be 'laminar'; a technical term meaning the flow is smooth. Flow will be laminar at low velocities but will become turbulent above some velocity, independent on the porous media and the permeating fluid. Darcy's law is not valid for turbulent flow. In most groundwater lowering applications flow can safely be assumed to be laminar [Cashman and. Preeney2001]. The only location where turbulent flow is likely to be generated is close to high flow rate wells pumping from coarse gravel aquifers. The implications of this flow to wells are discussed in for idealized condition, homogeneous, soils permeability depends primarily on the properties of the soil including the size and arrangement of the soil particles, and the resulting pore spaces formed when the particles are in contact. For example, consider an assemblage of billiard balls of similar size this is analogous to the structure of a high permeability soil (such as coarse gravel) where the voids (or pore water passages) are large, and the pore water can flow freely. Next, consider an assemblage of billiard balls with marbles placed in the spaces between the billiard balls this is analogous to a soil of moderate permeability because the effective size of the pore water passages are reduced. Finally, consider a structure with lead shot particles placed in the voids between the billiard balls and the marbles – the passages for the flow of pore water are further reduced; this simulates a low permeability soil. It is logical to infer from this analogy that the 'finer' portion of a sample dominates permeability [Cashman and. Preeney2001].

3. Governing equation

$$qV \frac{h}{\partial z} = K \frac{hA}{L} \frac{dh}{dz} - \phi \frac{dh}{dt} \quad \dots\dots\dots (1)$$

The governing equation express the rate of fluid flow in vertical column in homogeneous silty formation, the expressed equation developed is to express various deposition of fluid flow in soil and water environment There are numerous techniques for the direct determination of the void ratio and porosity, this also include permeability in the laboratory by inducing a flow of water through a soil sample, this approach is known as ‘permeameter’ testing Falling head test. An excess head of water is applied to the sample and the rate at which the head dissipates into the sample is monitored. Permeability is determined from the test results in a similar way to a falling head test in a borehole or observation well. These tests are suitable for soils of lower permeability (k_{1-10-4} m/s) when the rate of fall in head is easily measurable [Cashman and. Preeney2001].

$$\frac{dh}{dz} = S^1 h(z) - Sh(o) \quad \dots\dots\dots (2)$$

$$\frac{dh}{dz} = S^1 h(z) - Sh(o) \quad \dots\dots\dots (3)$$

$$\frac{dh}{dt} = S^1 h(t) - Sh(o) \quad \dots\dots\dots (4)$$

$$S^1 h(z) - (z) - qV [S^1(z) - S^1 h(o)] - K \frac{hA}{L} - a\phi [S^1 h(t) - Sh(t)] \quad \dots\dots\dots (5)$$

$$S^1(z) - h(o) = qVS^1 h(o) - qVho \quad \dots\dots\dots (6)$$

$$K \frac{hA}{L} h(z) - K \frac{hA}{L} S h(o) \quad \dots\dots\dots (7)$$

$$dh(z) - S h(o) \quad \dots\dots\dots (8)$$

Let $h(o) = 0$

We have

$$S^1(t) - qS^1 h(o) - K \frac{hA}{L} S^1 h(z) - a\phi h(t) \quad \dots\dots\dots (9)$$

$$h(z) = \frac{1}{S} \left[qVS^1 h(t) - K \frac{hA}{L} S^1 - a\phi S^1(t) \right] \quad \dots\dots\dots (10)$$

$$h(z) = \frac{1}{S^1} \left[qVS^1 h(t) - K \frac{hA}{L} S^1 - \alpha \phi(t) \right] \dots\dots\dots (11)$$

$$\frac{h^1(z) = qVS^1 h(t) - K \frac{hA}{L} h(z) - \alpha \phi S^1}{S} \dots\dots\dots (12)$$

$$h(z) = qV^2 h(t) - K \frac{hA}{L} h^1(z) - \alpha \phi h^1(t) \dots\dots\dots (13)$$

$$h(z) = qV^2 S^1 h(t) = \frac{K \frac{hA}{L} h^1(z) - \alpha \phi h^1(t)}{S} \dots\dots\dots (14)$$

$$h(z) = \left[qV^2 - K \frac{hA}{L} - \alpha \phi \right] h(t) \dots\dots\dots (15)$$

$$S^1 h(z) = \left[qV^2 - K \frac{hA}{L} - \alpha \phi \right] h(t) \dots\dots\dots (16)$$

$$h(z) = \frac{S^1 h(t)}{qVS^1 - K \frac{hA}{L} - \alpha} \phi \dots\dots\dots (17)$$

$$h(z) = \frac{S^1(z)}{qVS^1 - K \frac{hA}{L} - \alpha \phi S^1}$$

\dots\dots\dots (18)

The expressed model at [18] shows the rate of flow in a certain distance in the formation, the expression here implies that distance variation determine the rate of flow in the system as expressed in the model, the rate of flow are also influenced by hydraulic gradient of the fluid in the formations. The expressions here showcase the ability of the fluid to express it fluid velocity within a certain distance under a hydraulic gradient.

Furthermore, considering the boundary condition, we have the following

At $t = 0$ $h^1(o) = h(o) = 0$

$$qVS^1 - h(z) - K \frac{hA}{L} h(z) - \alpha \phi S^1 h(t) = 0 \dots\dots\dots (19)$$

$$h(z) = \frac{0}{qV - K \frac{hAS}{L} - \alpha\phi S} \dots\dots\dots (20)$$

Considering the following boundary condition when

At $t = 0$ $h^1(o) = h(o) = 0$

Apply the condition into this equation

Boundary values are determined, this to note the limit at which it can deposition in vertical column, definitely, the rate of porosity are expressed in the system, the integration of boundary values where found imperative because the limit at which the rate of flow under influence of soil porosity and void ratio are determined on the derived solution, the expression establishing boundary value became imperative because the rate of flow and the influence of his formation of the soil in the study location will be monitored.

$$qVS^1 - h(z) - qVho - q - K \frac{hA}{L} h(z) - K \frac{hA}{L} ho - h(z) - \alpha\phi h(t) - \alpha\phi ho + h(t) \dots\dots\dots (21)$$

$$qV(z) - K \frac{hA}{L} h(z) = qVS h(o) qho - K \frac{hA}{L} ho - h(z) - \alpha\phi ho \dots\dots\dots (22)$$

$$h(z) = \left[qVs + q + K \frac{hA}{L} + \alpha\phi \right] ho \dots\dots\dots (23)$$

$$h(z) = qVs - q - K \frac{hA}{L} - \alpha\phi ho \dots\dots\dots (24)$$

$$h(z) = \frac{\left[qVs - q - K \frac{hA}{L} - \alpha\phi \right] ho}{qVs - K \frac{hA}{L} - \alpha\phi} \dots\dots\dots (25)$$

Applying quadratic equation to determine denominator for the equation

$$qVs - K \frac{hA}{L} - \alpha\phi = 0 \dots\dots\dots (26)$$

$$S = \frac{-b \pm \sqrt{b^2 - 4ac}}{2ah} \dots\dots\dots (27)$$

Where $\alpha = qV$, $b = K \frac{hA}{L}$, $- c = \alpha\phi$

The concept of applying quadratic function is to ensure that the parameters function streamlined in the system, they should interact since they have relationship in the system, such condition are base on the structural development of the system, thus the rate of fluid as expressed mathematically, the parameters express their various function and which fit in within the system, it also establish their various influence on the flow path either in longitudinal path of tortuosity in soil and water environment.

$$S = \frac{-K \frac{hA}{L} \pm \sqrt{K \frac{hA^2}{L} - 4qV\alpha\phi ho}}{2qV} \dots\dots\dots (28)$$

$$\left[S_1 \frac{-K \frac{hA}{L} + \sqrt{K \frac{hA^2}{L} - 4qV\alpha\phi ho}}{2q} \right] \left[S_2 \frac{+K \frac{hA}{L} - \sqrt{K \frac{hA^2}{L} - 4qV\alpha\phi ho}}{2q} \right]$$

$$\ell \left[\frac{K \frac{hA}{L} - 4qV\alpha\phi ho}{2qV} \right] t^2 \left[K \frac{hA}{L} - \frac{\sqrt{K \frac{hA^2}{L} - 4qV\alpha\phi ho}}{2qV} \right] t \dots\dots\dots (29)$$

The Laplace inverse of the equation yield

$$h(z) = \left[\frac{qV}{z} + qV + K \frac{hA}{L} \right] ho \ell \left[K \frac{hA}{L} + \frac{\sqrt{K \frac{hA^2}{L} - 4qV\alpha\phi ho}}{2qV} \right] t \dots\dots\dots (30)$$

$$h(t) = \left[\frac{K \frac{hA}{L}}{t^2} ho \right] \left[K \frac{hA}{L} + \frac{\sqrt{K \frac{hA^2}{L} - 4qV\alpha\phi ho}}{2qV} \right] \ell \left[K \frac{hA}{L} \frac{\sqrt{K \frac{hA^2}{L} - 4qV\alpha\phi ho}}{2qV} \right]$$

$$\ell \left[\frac{K \frac{hA}{L} \sqrt{K \frac{hA^2}{L} - 4qV\alpha\phi ho}}{2qV} \right] t - \left[K \frac{hA}{L} \frac{\sqrt{K \frac{hA^2}{L} - 4qV\alpha\phi ho}}{2qV} \right] t \dots\dots\dots (31)$$

At this point $ho = 0 \quad t \neq 0$

For equation (30) at $t = 0 \quad h(o) = h(o)$, we have

$$ho = (qV + K \frac{hA^2}{L} - \alpha\phi)ho (1-1-1) = 0 = (q - K \frac{hA}{L} - \alpha\phi)$$

Hence $qV - K \frac{hA}{L} - \alpha\phi = 0$

Equation (31) becomes

$$h(z) = ho \left[\frac{qV}{t} + 2 \right] \left[K \frac{hA}{L} + \frac{\sqrt{K \frac{hA^2}{L} - 4qV\alpha\phi ho}}{2qV} \right] t \left[K \frac{hA}{L} \frac{\sqrt{K \frac{hA^2}{L} - 4qV\alpha\phi ho}}{2qV} \right] \dots \quad (32)$$

We recall that $e^x + e^{-x} = 2Cos x$, so that equation (32) can be expressed as:

$$h(z) = \left[K \frac{hA}{L} + 2 \right] ho Cos \left[K \frac{hA}{L} \frac{\sqrt{K \frac{hA^2}{L} - 4qV\alpha\phi ho}}{2qV} \right] t \dots \dots \dots \quad (33)$$

The express mode derived solution generated the final model in [33]. The expression are base on the fluid flow variation experienced in homogeneous silty formation in vertical soil column, the model express various condition in the derived solution, this showcase various variation of fluid flow rate in silty formation in vertical column, such condition were considered since the study location experienced slight variation in the flow rate of fluid within the intercedes of the soil. The model will definitely determine the flow rate of fluid in the formation as expressed in the system.

4. Conclusion

The rate flow fluid flow in soil and water environment obey some certain laws in fluid designed by several researchers, the law of Darcy has been express in various dimension, but the development of fluid flow in this dimension has not been expressed , the developed model applied this approach to monitor the rate fluid flow in silty formation, the expressed conceptual framework are to see the rate fluid flow in this approach, the developed mathematical expression were derived to monitor the rate of flow through in vertical soil column , the rate of solute may be expressed in this dimension but it depend on the type of solute transport in soil and water environment, meanwhile Darcy described the results of an experiment designed to study the flow of water through a porous medium. Darcy's experiment resulted in the formulation of a mathematical law that describes fluid motion in porous media. Darcy's Law states that the rate of fluid flow through a porous medium is proportional to the potential energy

gradient within that fluid. The constant of proportionality is the Darcy's permeability or hydraulic conductivity). Darcy's permeability is a property of both the porous medium and the fluid moving through the porous medium. In fact, Darcy's law is the empirical equivalent of the Navier-Stokes equations. Darcy's flow velocity for lamina flow is defined as the quantity of fluid flow along the hydraulic gradient per unit cross sectional area.

Reverences

- [1] Appleyard, S.J. (1993). Impact of storm water infiltration basin on groundwater quality, Perth Metropolitan region, Western Australia, *Environmental Geology*, vol. 21 no. 4, p.227-236.
- [2] Fisher, D., Charles, E. G., & Baehr, A. L. (2003). Effects of storm water infiltration on quality of groundwater beneath retention and detention basins, *Journal of Environmental Engineering*, vol. 129, no. 5, p. 464 - 471.
- [3] Liu, Y., Che, W. & Li, J. (2005). Monitor based evaluation of pollutant load from urban storm water runoff in Beijing, *Water Science and Technology*, vol. 52, no. 9, p. 191-197.
- [4] Lops, T.J., & Bender, D.A. (1998). Non point sources of volatile organic compounds in urban areas relative importance of land surfaces and air, *Environmental Pollution*, vol. 101, no. 2, p. 221-230.
- [5] Ku. I.F.H., Hagelin N.W., & Buxton H.T. (1992). Effects of urban storm runoff control on groundwater recharge in Nassau County, New York, *Groundwater*, vol. 30, no. 4, p. 507-514.
- [6] Mikkelsen, P.S., Weyer, G., Berry, C. Walden, Y., Colandini, V., Poulsen, S.,rotehusmann, D. & Rohlfig, R. (1994). Pollution from urban storm water infiltration, *Water Science and Technology*, vol. 29, no. 1-2, p. 291-302
- [7] Pitt, R. & Durrans S.R. (1995). Drainage of water from pavement structures, Alabama Department of Transportation, p. 253.
- [8] Pitt, R. (1996). Groundwater contamination from storm water infiltration, Ann Arbor Press, Chelsea, Michigan, p. 83-135.
- [9] Rahman, A., Lee, H.K. & Khan, M.A. (1997). Domestic water contamination in rapidly growing megacities of Asia : a case of Karachi, Pakistan, *Environmental Monitoring & Assessment*, vol. 44, p. 339-360.
- [10] World Health Organization (2006). Guidelines for drinking water quality recommendations, Chemical Facts Sheets, 3rd ed., vol. 1, p. 296-459.
- [11] Wilde, F.D. (1994). Geochemistry and factors affecting groundwater quality at three storm water management sites in Maryland, Report of Investigations no. 59, Maryland Geological Survey, Baltimore, Md., p. 201.
- [12] Zubair, A. (1993). Occurrence and properties of aquifer in Northern Karachi, M Phil thesis, Department of Geology, University of Karachi, Pakistan, p. 7-13 & 26.
- [13] Zubair, A. (1998). Groundwater pollution and its environmental impact on health in Karachi region Pakistan Ph D thesis, University of Ulster, Northern Ireland, UK, p. 113-123.
- [14] Malmquist P.A. & Hard S. (1981). Groundwater quality changes caused by storm water infiltration, Proceeding: 2nd international conference on Urban storm drainage, B. Yen (ed.), vol. 1, p. 89-97.

[15] Lops, T.J., & Bender, D.A. (1998). Non point sources of volatile organic compounds in urban areas relative importance of land surfaces and air, Environmental Pollution, vol. 101, no. 2, p. 221-230.

[16] Hathhorn, W. E., & Yonge, D.R. (1996). The assessment of groundwater pollution potential resulting from storm water infiltration BMP'S, Washington State Transportation Center (TRAC), (Final Report) Pullman, Washington, p. 16-17 & 35-42.

[17] Arif Z 2007 groundwater pollution resulting from storm water infiltration in Karachi principle investigator department of environmental sciences federal Urdu university of art, science & technology gulshan-e-iqbal campus, karachi-75300, Pakistan

[18] Eluozo S.N. Predictive model to determine the deposition of nickel and inhibition on Bacteriophage transport in shallow aquifers in coastal area of bonny, rivers state of Nigeria International Journal of Engineering and Technology Research Vol. 1, No. 3, April 2013, PP: 47 - 55